# A Computationally Efficient MASTeR-based Compressed Sensing Reconstruction for Dynamic MRI

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***Abstract-***State-of-the-art compressed sensing based algorithms recover sparse signals from under sampled incoherent measurements by exploiting their spatial as well as temporal structures. A compressed sensing based dynamic MRI reconstruction algorithm called MASTeR (Motion-Adaptive Spatio-Temporal Regularization) has shown great improvement in spatio-temporal resolution. MASTeR uses motion-adaptive linear transformations between neighboring images to model temporal sparsity. In this paper, a computationally efficient MASTeR-based scheme is presented that achieves the same image quality but in less time. The proposed algorithm minimizes a linear combination of three terms ($\ell$1-norm, total-variation andleast-square) for initial image reconstruction. Subsequently, least-square and $\ell$1-norm with ME/MC i.e., motion estimation and compensation are used to reduce the motion artifacts. The proposed scheme is analyzed for breath-held, steady-state-free-precession MRI scans with prospective cardiac gating.

***Keywords-***Compressed Sensing, Sparse Representation, Least Square Data Fitting, $\ell$1-norm regularization, Total Variation (TV) Minimization, Spatio-Temporal Regularization, Composite Problem.

## I. INTRODUCTION

Magnetic Resonance imaging (MRI) is a multipurpose, accurate, and non-invasivemedical imaging technique. MRI is commonly used in hospitals for medical diagnosis because it provides very detailed pictures of organs without exposing body to ionizing radiations. However, slow imaging speed is one of the several limitations to the use of MRI [i]. Slow speed poses many challenges to MRI, especially to dynamic cardiac MRI where the heart shows substantial movement as it goes from diastole to systole and vice versa. Dynamic MRI is based on the acquisition of series of frames to detect the motion of dynamic objects. Temporal as well as spatial resolutions of the cardiac MR image are related to the patient's ability to hold breath repeatedly. As many patients and children are unable to sustain breath-hold, poor image quality can be anticipated. Hence the imaging process must be accelerated to get rid of respiratory gating that makes scan longer.

For reduced imaging methods, reduction in reconstruction time can be achieved without increasing gradient performance. As reduced imaging takes only few phase encoding lines, the Nyquist criterion is violated. But exploiting the redundancy in images, artifact free images can be reconstructed. They either fill phase-encoding lines or unfold aliased images for this purpose [ii]. Temporal redundancy plays a major role in accelerating dynamic MRI, because most of structures in frames of a sequence are the same. In asame way, pixels can be predicted from neighboring pixels by exploiting spatial structures [iii]. Exploiting data redundancy reduces time required for artifact free image reconstruction. Hence increased temporal resolution at a given spatial resolution can be achieved and vice versa. Spatial and temporal structures of the images are frequently exploited to reconstruct images from under sampled k-space data.

Recent addition of compressed sensing (CS) methods [iv, v] to reduced-data imaging ensures the recovery of sparse signal from under sampled incoherent measurements under specific conditions. MASTeR is a recent addition to accelerated dynamic MRI techniques with multiple coils. This algorithm performs reconstruction in two steps namely image sequence estimation and estimation refinement. Our proposed scheme accelerates MASTeR by minimizing a linear combination of three terms – $\ell$1-norm regularization, least square data fitting, and total variation – for initial reconstruction from k-space.The rest of the paper is organized as follows. Section II describes compressed sensing theory, dynamic MRI reconstruction, Motion-Adaptive Spatio-Temporal Regularization, Optimization algorithms, and the proposed accelerated MASTeR scheme. In section III, experimental setup is described. This section also discusses composite problems and related solvers. Results are presented in section IV while section V concludes the paper.

## II. Theoretical Background

### A. Compressed Sensing

Compressed sensing or compressive sensing (CS) is a new sampling theory for efficiently acquiring and reconstructing a signal from under determined linear systems [vi, vii]. The structured signals are acquired at very low sampling rate as compared to Nyquist rate without quality degradation. CS removes the need to acquire large amount of data and then compress it, by sampling the signal in space using non-adaptive sampling techniques that compress the signal and reconstruct using optimization schemes [viii]. CS claims the accurate reconstruction if: (a) the desired image is sparse in a known transform domain, (b) measurement are incoherent in that domain, (c)non-linear reconstruction promote the desired structure while simultaneously preserving the fidelity of the measurements [ix, x].

CS inherently suits magnetic resonance imaging (MRI) as medical images are sparse in some transform domain and MRI scanners store encoded samples of spatial frequency instead of pixels [xi]. Incoherent (noise like) under-sampling is another essential feature of CS reconstruction, and hence MR acquisition setup is designed to exploit it. The data acquisition and image reconstruction process as a whole can be further accelerated by use of parallel imaging with CS methods. It exploits the spatial sensitivity which is naturally present in the coil array to remove (time consuming) spatial encoding. Coil sensitivities also help to provide aliasing free image reconstruction [xii].

The essence of recovery procedure is to solve an optimization problem. Simplest optimization problem of the MR image recovery is the Least Squares (LS) method that reconstructs images without information about any specific structure. If matrix $A$ is full column rank, $x$ can be estimated by solving the following [xiii]:

$$\underset{x}{minimize} \|\boldsymbol{Ax}-\boldsymbol{b}\|_2^2 \tag{1}$$

where $\boldsymbol{x}$ denotes the MR image to be reconstructed, $\boldsymbol{b}$ denotes the k-space measurement vector for $\boldsymbol{x}$, and $\boldsymbol{A}$ is the encoding matrix that contains Fourier coefficients weighted by coil sensitivities.

For under determined systems using only least squares is not reliable [vi]. MR images exhibit sparsity in various transforms like wavelet or finite difference. A new regularization term can be added to (1) to incorporate sparsity in the recovery process. Using $\ell 1$-norm as regularization term (1) becomes:

$$\underset{x}{minimize} \|\boldsymbol{Ax}-\boldsymbol{b}\|_2^2 + \boldsymbol{\beta}\|\boldsymbol{\Psi x}\|_1 \tag{2}$$

$\ell 1$-regularization term promotes sparsity in the solution while $\ell 2$-regularization keeps data consistency. In (2), $\beta > 0$ is regularization parameter that controls the fidelity of reconstruction towards measurement whereas $\Psi$ could be a sparse transform, e.g. wavelet. Equation (2)determines the compressible solution thus enforcing data consistency. In $\ell 1$-regularization, many small coefficients tend to carry large penalty as compared to fewer large coefficients. This makes the solution sparse by suppressing (many) small coefficients [xiv].

Convex optimization programs minimize normalization of sparse signals with certain data-fidelity constraints. Sparse MRI [iv] exploits spatial sparsity in image domain for angiography; and in total variation or wavelet domain for cardiac and brain MRI respectively. Sensitivity Encoding (SENSE) based methods like CS-SENSE [xv] and Sparse-SENSE [vi] combine parallel imaging with CS.

### B. Dynamic MRI Reconstruction

Dynamic MRI reconstruction is challenging also due to time varying nature of the objects and spatio-temporal tradeoff [xvi]. In case of dynamic MRI, temporal sparsity plays an important role. Exploiting temporal structures in a sequence of frames could produce good results. Fourier transform along temporal dimension sparsifies the quasi-periodic signals effectively like cardiac cine while Karhunen-Loeve transform (KLT) outperforms for non-periodic signal like fMRI [xvii].

Wavelet transform shows better results when used as spatially sparsifying transform because it decomposes a spatial structure into a low frequency component(approximate image) and a high frequency component(detail image) that are sparse [xviii, xix]. As temporal variations are linked with spatial domain variations in different frames and neighboring pixels, a transform model that can deal with both simultaneously provides effective model of dynamic MRI reconstruction. Many state of the art methods like kt-FOCUS [xx] and kt-sparse [vii] etc. tried to address this issue. Kt-FOCUS models inter-frame motion against entirely sampled reference frame. If reference frame is not available it can be obtained from averaging of measurements in k-t space of a complete phase e.g. diastole or systole. Filling up the k-space of reference frame requires as many cycles as for all frames and reconstruction quality of Kt-FOCUS directly depends upon reference frame. Motion-Adaptive Spatio-Temporal Regularization(MASTeR)published in [xxi] eliminated the need of reference frame through considerable improvement in spatio-temporal regularization.

MASTeR [xxi] is an excellent addition to dynamic MRI techniques with array coil. In this technique, temporal sparsity is modeled by inter-frame motion while spatial sparsity is modeled by wavelet transform. A motion adaptive transform links neighboring pixels and interpolates pixel intensities. Inter-frame motion decides about the new locations of interpolated pixels.It considers dynamic MR images as a sequence of video frames and an image is estimated through motion-adaptive interpolation of neighbor images. The difference of original and predicted image is known as motion-compensated residual. This residual image is often a sparse image.

The reconstruction process of MASTeR consists of two steps [xxi]. In first step, image sequence is estimated without any motion information. In second step, image estimates are iteratively refined by exploiting inter-frame motion. Numerous video compression techniques exploit inter-frame prediction as a necessary component [xxii]. Motion is estimated between a reference frame and its preceding and subsequent frames. These residual values are used to reconstruct images. Second step is repeated many times for better quality. Motion adaption scheme of MASTeR overcomes the difficulty of generating a fully sampled reference image that takes as much time as collecting samples for the complete sequence. The algorithm solves a convex optimization problem whose computational cost depends upon data mismatch, sparsity of images in spatial domain and sparsity of residuals in temporal domain.

The algorithms that solve the optimization problem play an important role in the quality and speed of image reconstruction. ℓp-quasinorm ($p < 1$) regularization optimization provides better compression ratio but no global minima is guaranteed [xxiii]. Homotopic nonconvex ℓ0-minimization is faster than ℓp and could converge globally under desired parameters [xxiv]. ℓ1-regularization provides global convergence with less computational time. TV regularization when used with ℓ1-regularization improves image quality in lesser time [xxiii].If the image is sparse, ℓ1-regularization works better but for piece wise linear objects Total variation (TV) provides accurate results [xxv].

MASTeR uses ℓ1-regularization for solving the optimization problem to stimulate sparsity in both spatial as well as temporal domains. Many state of the art ℓ1-norm minimization algorithms are available but either they are not accurate or take too much time to solve the problem. NESTA [xxv] as ℓ1-norm minimization solver leads state-of-the-art algorithms in both accuracy and computational time. NESTA also added continuation (NESTA+CT) inspired from homotopy techniques to further reduce the computational time with reasonable accuracy but time is still an important issue in making the MASTeR suitable for practical applications.

### C. *Accelerated MASTeR*

Our proposed scheme contributes towards accelerating the MASTeR. It is based on the fact that false detection of fine-scale (detail) wavelet components may render small high-frequency oscillatory artifacts in the reconstruction image when using ℓ1-regularization is used. For the recovery of (approximately) sparse signals, ℓ1-regularization outperforms but for piece-wise smooth objects TV normalization surpasses ℓ1-regularization [xxv, xxvi]. Under the assumption that the objects in the image either consist of constant or mildly varying areas, it is useful to include TV penalty, even when using other sparsifying transform e.g. DFT, wavelet etc.[iv, xxvii]. It is just like requiring the image to be sparse simultaneously both in transform domain as well as finite-difference domain. It helps to reduce streaking artifacts and noises. The source of inspiration to accelerate MASTeR is the high convergence rate of composite (TV+L1) optimization solvers. So quality MRI reconstruction can be obtained in lesser time.

The proposed scheme minimizes a linear combination of three measures, viz.ℓ1-norm regularization, least square data fitting, and total variation for preliminary image reconstruction from k-space. The ℓ1-norm in the objective function is a critical parameter but we cannot ignore the importance of TV in sparse reconstruction. By adding the TV term (2) becomes:

$$\underset{x}{\text{minimize}} \sum_i \|\boldsymbol{A_i x_i} - \boldsymbol{b_i}\|_2^2 + \boldsymbol{\alpha}\|\boldsymbol{x_i}\|_{\boldsymbol{TV}} + \boldsymbol{\beta}\|\boldsymbol{\Psi x_i}\|_1 \quad (3)$$

where α is also positive regularization parameter and such an optimization problem is known as composite problem [xxviii]. To further refine the frames, motion adaption scheme of MASTeR is used, that solves optimization problem following the dynamical system.

$$\underset{x}{\text{minimize}} \sum_i \|\boldsymbol{A_i x_i} - \boldsymbol{b_i}\|_2^2 + \boldsymbol{\tau}\|\boldsymbol{F_{i-1} x_{i-1}} - \boldsymbol{x_i}\|_1 + \boldsymbol{\gamma}\|\boldsymbol{B_{i-1} x_{i-1}} - \boldsymbol{x_i}\|_1 \quad (4)$$

where τ and γ are regularization parameters and $\boldsymbol{F}_{i\text{-}1}$, $\boldsymbol{B}_{i\text{-}1}$ are forward and backward motion operators respectively. Motion adaptive transforms are constructed from these operators that in turn exploit inter-frame motion for describing image sequence as residual values. The results show that the proposed scheme namely Accelerated MASTeR is computationally more efficient than Standard MASTeR and simultaneously maintains the same image quality.

## III. EXPERIMENTAL SETUP

Breath-held, steady-state free precession (SSFP) MRI scans with prospective cardiac gating were used in the experiments conducted by Asif et al. [xxi]. In such experiments, fully sampled k-space data with multiple coils was down sampled. The images were subsequently reconstructed from this down-sampled k-space data. The performance of our proposed scheme was evaluated by reconstructing cardiac MRI datasets for multiple reduction factors and the results were compared with those of standard MASTeR.

A short-axis MRI scan was obtained using a GE 1.5T Twin Speed scanner that is equipped with a 5-element cardiac coil [xxi]. These images are depicted in Fig. 1. The scan was performed with the following parameters: TE: 2.0 ms, TR: 4.1 ms, flip angle: 45,FOV: 350 x 350mm, slice thickness: 12 mm, 8 views per segment, 224 phase-encoding lines, 256 read-out samples, and 16 temporal frames. A separate scan known as prescan was performed with identical parameters to emulate the estimation of sensitivity maps. k-spacedata from each coil was then down-sampled by a factor of 2. The down sampled data is subsequently used to get smoothed images for each coil using inverse Fourier transform. Each smoothed coil image was divided by the root-sum-of-squares of all coil images to estimate sensitivity maps.

A two-chamber view cine MRI scan was obtained using a Philips Intera 1.5T scanner equipped with a 5-element cardiac coil [xxi]. These images are depicted in Fig. 3. These scans were carried out with the following parameters: TE: 2.2 ms, TR: 4.4 ms, flip angle: 45, slice thickness: 8 mm, 240 phase-encoding lines, 200 read-out samples, and 16 temporal frames. To maintain fully registered sensitivity profile, they were estimated from full data. This shows positive bias in SNR of measurements and eliminate expected errors that might be caused by misregistration. 2D Cartesian down sampling pattern was according to standard Gaussian distribution with selection of 8 low frequency phase encoding lines around the centre of k-space and remaining in random form high frequency.

Our research work presents a new framework based on the use of FCSA and NESTA combination for dynamic MRI reconstruction. Standard MASTeR [xxi] uses NESTA+CT for initial image estimates that lead to $\ell$1-minimzation algorithms in terms of accuracy and time. Despite of its efficiency, NESTA still takes too much time as compared to algorithms solving composite problem. Nevertheless we cannot ignore the continuation patch of NESTA that plays vital role in refining image for higher resolutions. The proposed algorithm performs initial image reconstruction using modified fast composite splitting algorithms (FCSA) – a recently proposed algorithm to efficiently solve the composite problem [xxiv] modeled as (3).Two algorithms namely the MSA (multiple splitting algorithms) and its fast version FaMSA [xxix] also claim to efficiently solve the composite problem modeled in (3). These algorithms assume that all convex functions are smooth; hence it makes them unable to directly solve the problem as we have to smooth the non-smooth functions before applying them. This increases the computational complexity and affects their time efficiency.

FCSA solves the composite problem by first decomposing the original problem into $\ell_1$ and TV norm regularization sub problems respectively. Then, these two sub problems are efficiently solved by existing techniques TVCMRI [xxvi] and RecPF [xxx]. The fast convergence of FCSA is borrowed from FISTA [xxxi]. Finally, the reconstructed image is obtained from the weighted average of solutions from two sub problems in an iterative framework. Here wavelet transform with TV penalty was used as spatial sparsifying transform to make the image sparse in both the specific transform and finite-difference domain at the same time. Such transforms contribute towards better SNR in short time. The regularization parameters $\alpha$ and $\beta$ were set as $1\times10^{-5}$ and $3.5\times10^{-5}$. The number of iterations was set to 10. The step size in FCSA is designed according to inverse of Lipchitz constant.

FCSA's wavelet transform compilation module was replaced by Matlab wavelet tool box to make it more time efficient and suitable for large scale problems. For motion adaptation iterations, inter-frame motion was estimated using phase-shifts of complex wavelet transform (CWT) coefficients. NESTA+CT was used to solve $\ell$1-regularization terms of (4) in motion adaption step. It is based on homotopy technique and hence shows great improvement in quality during motion refine iterations. Our scheme utilizes the best features of FCSA and NESTA+CT. As NESTA takes too much time in initial image estimate so we replaced it with FCSA's modified version. Initial image is estimated with same accuracy as NESTA but in lesser time, and then NESTA+CT contribute to refine image sequence. Motion compensation technique from standard MASTeR algorithm is also utilized for minimizing motion related artifacts.

To compare the results of our proposed scheme with standard MASTeR, we performed reconstruction for a number of reduction factors (N/M). Standard MASTeR performs initial image estimate from k-space data by solving optimization problem through $\ell$1-minimization. It uses NESTA toolbox as $\ell$1-minimization solver and Wavelet transform as spatial sparsifying transform. Regularization parameters were selected to minimize RMS error between original and reconstructed images. Tolerance was set to 1e-6; maximum internal iterations for NESTA were set to 50 and 5 motion refine iterations were performed using NESTA+CT. Overall setup provides a fair comparison. All of our simulations were carried out in MATLAB on a personal computer with a 2.4 GHz Intel Core i3 processor and 2GB RAM.

## IV. RESULTS AND DISCUSSIONS

**Short Axis Dataset:** Fig. 1 demonstrates the comparison of our proposed scheme with MASTeR for the reduction factors of 6 and 10. Frames 1, 3 and 9 are shown from left to right. Fig. 1(a) represents the frames reconstructed from full k-space data. Fig.1 (b) and 1(c) show the images reconstructed from proposed scheme and MASTeR respectively. Reconstruction at the reduction factor of 10 is displayed in Fig. 1(d) and (e). Fig. 2 shows the cropped and zoomed region of interest (ROI) of the frames in Fig. 1. Here heart is the region of interest where most of the changes occur. Visual comparison describes that accuracy of both schemes is approximately same.

**Two-chamber MRI Dataset:** Fig. 3 presents the visual comparison of proposed scheme with MASTeR for Two-chamber MRI dataset at reduction factors of 6 and 10. Images reconstructed from full k-space data are shown in Fig. 3(a). Frames 1, 3 and 9 are shown from left to right. Fig. 3(b) and 3(d) presents the results of proposed scheme at reduction factors of 6 and 10 respectively. Reconstruction results of MASTeR are shown in Fig. 3(c) and 3(e) for reduction factor 6 and 10 respectively. Cropped and zoomed ROI of two-chamber MRI data set are displayed in Fig. 4.

TABLE I tabulates the time consumed by the two methods for reconstruction of 16 MRI frames. Fig. 5 shows that our proposed scheme outperforms standard MASTeR for all reduction factors. A quantitative comparison of both schemes for different reduction factors is presented in Fig. 6. TABLE II contains the performance evaluation based on Signal-to-Error ratio in dB, using:

$$SER = 10log_{10}\frac{\|x\|_2^2}{\|x-\hat{x}\|_2^2} \qquad (5)$$

where $x$ and $\hat{x}$ are images constructed from full k-space data and reconstructed image from under sampled k-space data respectively. Blue bars denote SER of standard MASTeR and auburn bars denotes proposed scheme i.e., accelerated MASTeR. Graph shows that both schemes have almost the same accuracy for all reduction factors, which verifies the validity and usefulness of our acceleration scheme.

Source of improvement: The main difference between Standard MASTeR and Accelerated MASTeR is of initial image estimation. Standard MASTeR solves ℓ1-regularization problem of Eq. 2 to recover initial image estimate while Accelerated MASTeR minimizes a linear combination of three terms, corresponding to least square data fitting, total variation and ℓ1-norm regularization. Use of TV penalty with sparsifying transform in (3) makes image to be sparse in both specific transform and finite difference domain at the same time. Finite-difference transform is considered as computing some sort of fine-scale wavelet transform. Hence it mitigates the small high-frequency oscillatory artifacts that appear in reconstruction due to the false detection of fine-scale wavelet components. In quantitative context, combination of ℓ1 and TV regularization contribute towards better SER in lesser time as compared to just using ℓ1-minimization.

TABLE I
PROCESSING TIME COMPARISON

| Reduction Factor | Standard MASTeR (sec) | Accelerated MASTeR (sec) |
|---|---|---|
| 2 | 1554.15 | 1032.11 |
| 4 | 1791.02 | 1139.07 |
| 6 | 1861.71 | 1259.28 |
| 8 | 1846.45 | 1289.91 |
| 10 | 1882.06 | 1314.11 |
| 12 | 1891.21 | 1344.23 |
| 20 | 1949.29 | 1369.59 |

TABLE II
SER COMPARISON

| Reduction Factor | Standard MASTeR (dB) | Accelerated MASTeR (dB) |
|---|---|---|
| 2 | 1067 | 1069 |
| 4 | 652 | 646 |
| 6 | 486 | 474 |
| 8 | 370 | 359 |
| 10 | 275 | 269 |
| 12 | 230 | 224 |
| 20 | 75 | 90 |

Computational complexity of FCSA for each iteration is O (plog(p)) while that of NESTA is $O(p^2)$ where p is the pixel number in the reconstructed image. FCSA approaches to $\epsilon$- optimal solution in $O(1/\sqrt{\epsilon})$ iterations as compared to NESTA that requires $O(1/\epsilon)$ iterations. Here $\epsilon$ serves as boundary condition for reconstruction error. Its fast convergence outperforms the state-of-art methods like RecPF and TVCMRI [xxvi]. RecPF itself is about 10 times faster than NESTA [xxxii].

We used set and search recurring process to determine the optimum values of regularization parameters for minimum RMS error. The step size of FCSA is designed to be inverse of the Lipchitz constant, so larger values may contribute to faster version of algorithm [xxiv].

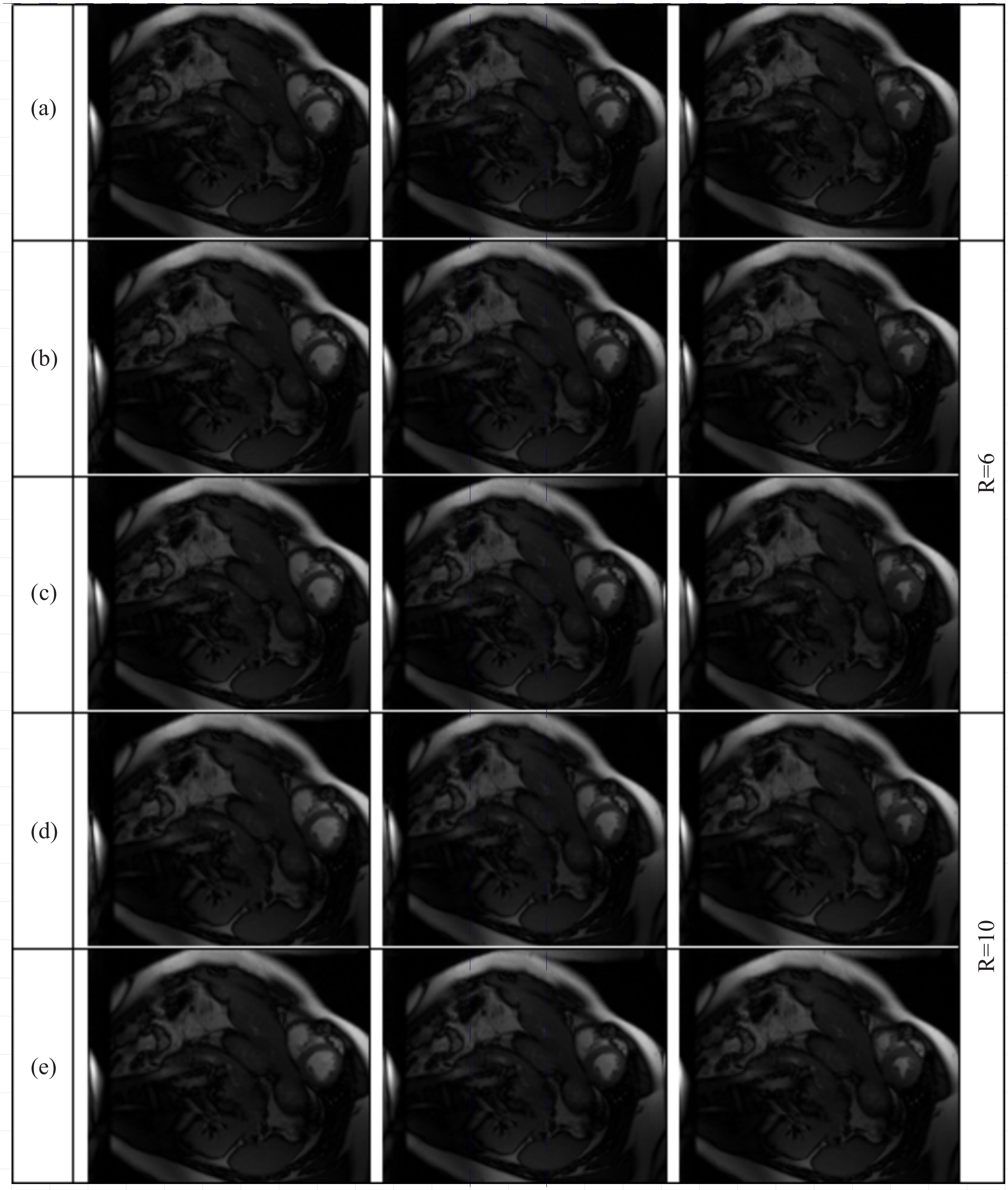


Fig. 1. Comparison of Accelerated MASTeR and Standard MASTeR for short axis MRI scan: frames 1, 3 and 9 from left to right. (a) Images constructed from full k-space data. (b) Shows the reconstruction through Accelerated MASTeR and (c) using Standard MASTeR at reduction factor of 6. (d) Shows the reconstruction through Accelerated MASTeR and (e) using Standard MASTeR at reduction factor of 10.

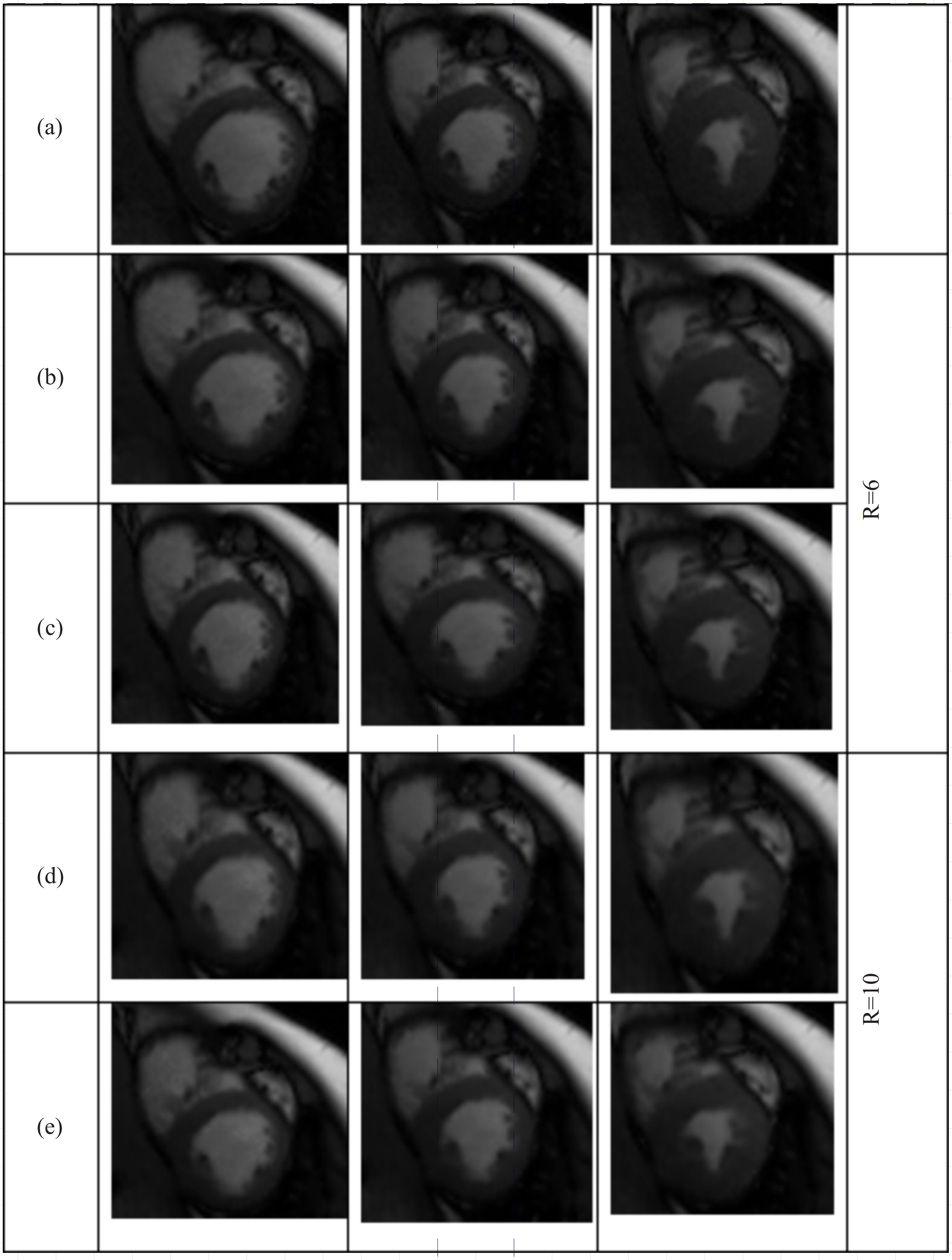


Fig. 2. Comparison of Accelerated MASTeR and Standard MASTeR for ROI of short axis MRI scan: frames 1, 3 and 9 from left to right.(a)Cropped and zoomed ROI constructed from full k-space data.(b) The reconstruction through Accelerated MASTeR and (c) using Standard MASTeR at reduction factor of 6. (d) The reconstruction through Accelerated MASTeR and (e) using Standard MASTeR at reduction factor of 10.

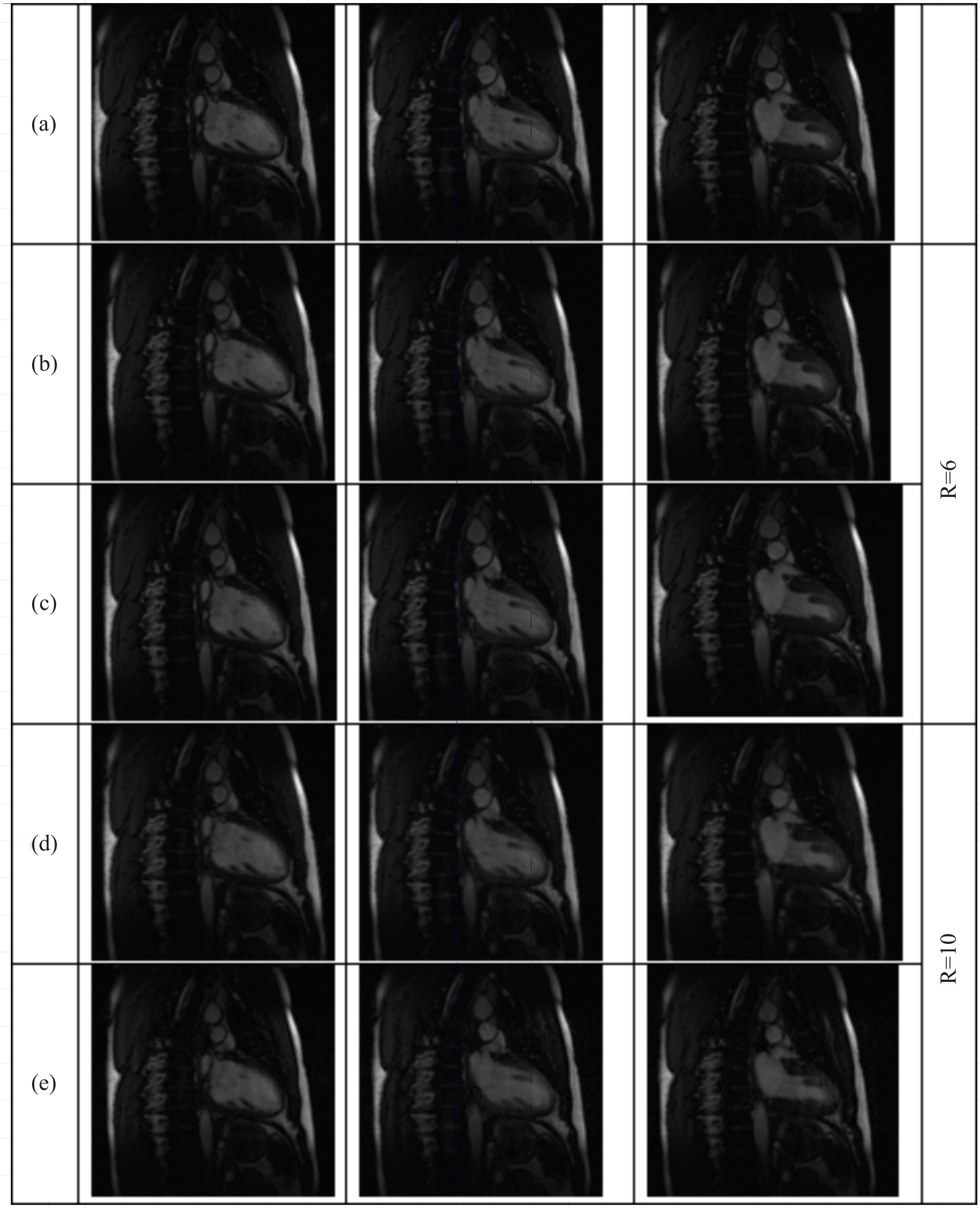


Fig. 3. Comparison of Accelerated MASTeR and Standard MASTeR for two chamber MRI scan: frames 1, 3 and 9 from left to right.(a) Images constructed from full k-space data.(b)shows the reconstruction through Accelerated MASTeR and (c) using Standard MASTeR at reduction factor of 6. (d) Shows the reconstruction through Accelerated MASTeR and (e) using Standard MASTeR at reduction factor of 10.

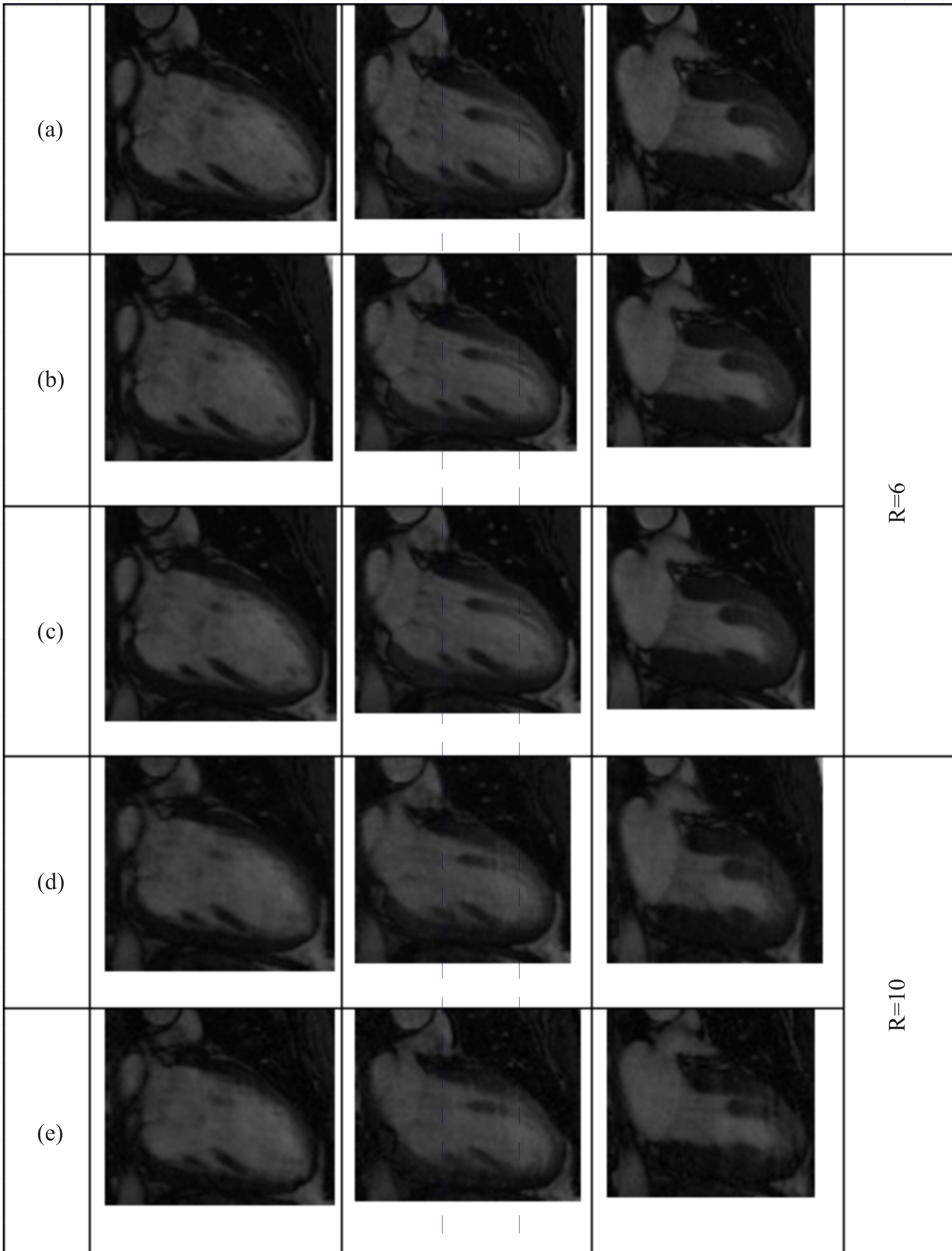


Fig. 4. Comparison of Accelerated MASTeR and Standard MASTeR for ROI of two chamber MRI scan: frames 1, 3 and 9 from left to right.(a)Cropped and zoomed ROI constructed from full k-space data.(b) The reconstruction through Accelerated MASTeR and (c) using Standard MASTeR at reduction factor of 6. (d) The reconstruction through Accelerated MASTeR and (e) using Standard MASTeR at reduction factor of 10.

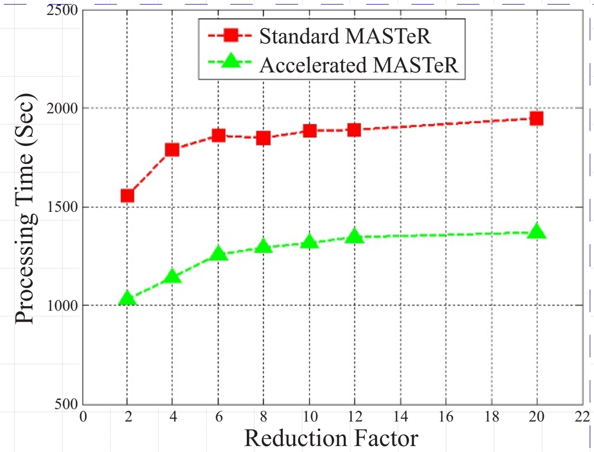


Fig. 5. Processing time comparison of Accelerated MASTeR (green ▲) with Standard MASTeR (red ■)

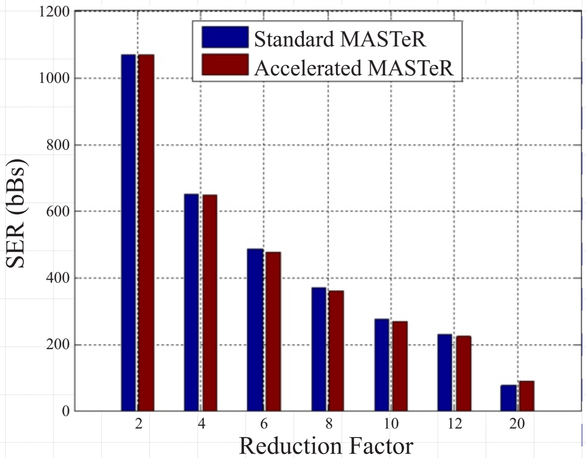


Fig. 6. SER Comparison of Accelerated MASTeR (Auburn) with Standard MASTeR (blue)

The experimental results described above validate the efficiency of our proposed scheme for dynamic MRI reconstruction. The main contribution of this paper is to efficiently recover dynamic MRI sequence from down sampled k-space data. The Accelerated MASTeR takes the advantage of both algorithms. It provides the initial image estimate of reasonable quality by using FCSA in short time and then solves (4) to improve it through NESTA+CT for forward and backward motion estimation and compensation borrowed from MASTeR.

A major drawback of FCSA is its limitation for high level accuracy. It does not show remarkable improvement in quality during motion adaption where inter frame motion refining operation is performed. NESTA+CT outperforms here due to continuation inspired from homotopy techniques. NESTA+CT solves sequence of problems with decreasing value of smoothing parameter and use the intermediate solution as a warm start for the next problem. Continuation has been shown to be a very successful tool for increasing the speed of convergence, particularly when dealing with large-scale problems and high dynamic range signals [xxxiii].

## V. CONCLUSIONS

The proposed algorithm is compared with MASTeR which has already proven superior than k-t FOCUSS with ME/MC and other recently proposed fast algorithms. The proposed scheme performs initial reconstruction in lesser time without compromise on quality.Combination of total-variation and $\ell_1$-norm for initial image reconstruction contribute towards better SER in lesser time for dynamic MRI. This provides better initial guess for later motion adaption steps enabling it to reconstruct dynamic MRI in lesser time as compared to MASTeR. We compared proposed scheme with MASTer for number of reduction factors. Under identical settings of recovery framework, the proposed scheme can achieve the same reconstruction accuracy as that of MASTeRbut consuming approximately 30% lesser time. A good future work could be the extension of FCSA that takes advantage of homotopy techniques to attain higher level of accuracy in short time, so that faster algorithm can be developed.

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